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ANALYTICAL APPROXIMATIONS

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## Analytical Approximation

**Chi-Square Integral:** To better than .0004 over $0 \leq \chi^2 \leq 3$ for $m = 3$,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{.6084}{\left[1+.1567\eta+.0564\eta^2+.0039\eta^3\right]^4};$$

$$\eta = \sqrt{\frac{m}{2}} \ln\left(\frac{r}{\chi^2}\right).$$

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## Analytical Approximation

Chi-Square Integral: To better than .0004 over

$0 \leq \chi^2 \leq 4$ for $m = 4$,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{.5940}{\left[1+.1627\eta+.0604\eta^2+.0069\eta^3\right]^4};$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)}.$$

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## Analytical Approximation

**Chi-Square Integral:** To better than .0005 over $0 \le \chi^2 \le 5$ for $m = 5$,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{.5841}{\left[1+.1671\eta+.0626\eta^2+.0097\eta^3\right]^4};$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)}.$$

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## Analytical Approximation

Chi-Square Integral: To better than .0005 over

$0 \leq \chi^2 \leq 7$ for $m = 7$,

$$F_m(\chi^2) = \frac{1}{2\,\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2}\, d(t^2)$$

$$\doteq \frac{.5711}{\left[1+.1731\eta+.0647\eta^2+.0143\eta^3\right]^4}\ ;$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)}\,.$$

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## Analytical Approximation

Chi-Square Integral: To better than .0006 over $0 \leq \chi^2 \leq 8$ for $m = 8$,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{.5665}{\left[1+.1752\eta+.0653\eta^2+.0162\eta^3\right]^4};$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)}.$$

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# Analytical Approximation

Chi-Square Integral: To better than .0006 over $0 \leq \chi^2 \leq 9$ for $m = 9$,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{.5627}{\left[1+.1770\eta+.0657\eta^2+.0178\eta^3\right]^4};$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)}.$$

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Analytical Approximation

Chi-Square Integral: To better than .0006 over $0 \leq \chi^2 \leq 2$ for $m = 2$,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{.6321}{\left[1+.1474\eta+.0483\eta^2+.0010\eta^3\right]^4};$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)}.$$

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## Analytical Approximation

Chi-Square Integral: To better than .0005 over $0 \leq \chi^2 \leq 6$ for $m = 6$,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{.5768}{\left[1+.1704\eta+.0640\eta^2+.0121\eta^3\right]^4},$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)}.$$

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# Analytical Approximation

Chi-Square Integral: To better than .0006 over $0 \leq \chi^2 \leq 10$ for $m = 10$,

$$P_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{.5595}{\left[1+.1786\eta+.0659\eta^2+.0194\eta^3\right]^4} ;$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)} .$$

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Analytical Approximation

Chi-Square Integral: To better than .0009 over

$0 \leq \chi^2 \leq 30$ for $m = 30$,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{.5344}{\left[1+.1911\eta+.0661\eta^2+.0339\eta^3\right]^4};$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)}.$$

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